UDK: 627.841; 628.15; 626.82

A. D. TAGHIYEVA

DEVELOPMENT OF MODEL OF OPTIMAL CONTROL OF WATER SUPPLY SYSTEM

Abstracts: This paper considers the problem of optimal control of branched water supply systems. To control the system, the problem of optimal distribution of products is developed whereas non-linear programming problems are applied. We consider a system for providing products, consisting of magistral and distribution pipelines, taking products from the magistral pipeline. Each distribution line has many warehouses. Products are taken into the system using the main intake facility and transferred between the warehouses using intermediate distribution facilities. To eliminate the deficiencies in management, tasks are set to determine the necessary intensities of product supply in the facilities, allowing timely provision of consumers with the necessary volume of products, to minimize losses, product discharges of facilities in the system during a certain control period.

Keywords: *water supply system, water supply line, water supply point, optimal control, water consumption, optimization criteria.*

Introduction

The introduction of modern management systems in the water management provides enterprises with an unprecedented opportunity to control and manage all aspects of water intake, transportation, and distribution from a centralized management system. A new method for optimizing this system in real time is proposed, formulated as an integer quadratic programming problem. The proposed method for solving this problem is very successful in achieving an almost optimal solution. Modern water management enterprises should be a single system, operating with information-computing system [1]. Benefits resulting from these actions can include improving the quality of water supply by reducing water loss, minimizing energy costs, and increasing system performance without compromising operational reliability. The real-time demand management strategy is applied to water supply enterprises to reduce the target cost function as low as possible [2].

Characteristics of the object

In this work, we consider a product supply system, which consists of a madistral line (ML) and K the number of distribution lines (DL) taking products from the ML. In each DL there are J_k number of warehouses (QW). Products are taken into the system with the help of the main fence (C-00) and transferred between the warehouses with the help of intermediate distribution structures (C-*kj*, $k=\overline{0,K}$, $j=\overline{1,J_k}$). Excessive products can be removed from the system using emergency facilities (C-kJk) at the end of each line. On each j-t section of the q line there are $İ_{kj}$ the number of consumers (T). To control such systems, the necessary intensities of product supply at each structure are calculated, which are supported by an automated control system [3]. But the incorrect calculation of these intensities causes large losses, product discharges, excessive switching of equipment and untimely provision of consumers. As a result of this, the system management efficiency is reduced.

Statement of the problem

To eliminate the above disadvantages, the following statement of the problem of determining the necessary intensities is given:

It is required to find such intensities of supply of products in structures that can provide consumers with the necessary volume of products in a certain manner, minimize losses, product discharges and the number of changes in the operating modes of structures in the system during the control period (t_0, T] [4].

We divide the period (t_0, T] into Z the number of intervals (t_{z-1}, t_z], on each of which the intensities are almost constant, (t, T]= $\bigcup_{z=1}^{Z}(t_{z-1},t_z]$. We use the following notation: V_{kj}^z, $k=\overline{0,K}$, $j=\overline{1,J_k}$ – reserves of products in warehouses during the interval (t_{z-1}, t_z]; Q_{kj}^z, $k=\overline{0,K}$, $j=\overline{0,J_k}$ – intensities of supply of products in line facilities; $q^z_{kji}, k=\overline{0,K}, j=\overline{1,J_k}, i=\overline{1,I_{kj}}$ – intensities of consumption of products at points of consumption; C_{kJ_k}, $k=\overline{0,K}$ – penalties for dumping a unit volume of production; $C^{'}_{kj}$ – penalties for losses of a unit volume of production in sections; C_{kj}, $k=\overline{0,K}$, $j=\overline{0,J_k}$, $(kj)\neq(00)$ – penalties for supplying products in intermediate structures with a changed intensity. $C^{'}_{kj}$ for supplying products with an intensity different from, q^{Tz}_{kji}. Product losses in the areas taken proportional to stock products in which $Q^{pot,z}_{kj}=L_{kj}V^z_{kj}$, $k=\overline{0,K}$, $j=\overline{0,J_k}$, where L_{kj} – proportionality factor.

As the objective function, the sum of the costs associated with the discharge of products from the automation system $Q^z_{kJ_k}$, with consumption, losses in the areas $L_{kj}V^z_{kj}$ and, with the squares of the flow rate change in C-kj ($Q^z_{kj}-Q^{z-1}_{kj}$) and T-kj ($q^{Tz}_{kji}-q^z_{kji}$):

$$f=\sum_{z=1}^{Z}\sum_{k=0}^{K}C_{kJ_k}Q^z_{kJ_k}\Delta t_z+$$

$$+\sum_{z=1}^{Z}\sum_{k=0}^{K}\sum_{j=1}^{J_k}C'_{kj}L_{kj}V^z_{kj}\Delta t_z+$$

$$+\sum_{z=1}^{Z}\sum_{k=0}^{K}\sum_{j=0}^{J_k-1}C_{kj}\left[\left(Q^z_{kj}-Q^{z-1}_{kj}\right)\Delta t_z\right]^2+$$

$$+\sum_{z=1}^{Z}\sum_{k=0}^{K}\sum_{j=1}^{J_k}\sum_{i=1}^{I_{kji}}C_{kji}\left[(q^z_{kji}-q^{Tz}_{kji})\Delta t_z\right]^2\to\min.$$

Denote

$$C^z_{00}=C_{00}\Delta t_z, C^z_{kJ_k}=C_{kJ_k}\Delta t_z,$$

$$C^{pot,z}_{kj}=C^{'}_{kj}L_{kj}\Delta t_z, C^z_{kj}=C_{kj}\Delta t_z{}^2,$$

$$C^z_{kji}=C_{kji}\Delta t_z.$$

Then, the objective function is transformed to the form:

$$f=\sum_{z=1}^{Z}\sum_{k=0}^{K}C^z_{kJ_k}Q^z_{kJ_k}+\sum_{z=1}^{Z}\sum_{k=0}^{K}\sum_{j=1}^{J_k}C'^z_{kj}Q^{it,z}_{kj}+$$

$$+\sum_{z=1}^{Z}\sum_{k=0}^{K}\sum_{j=0}^{J_k-1}C^z_{kj}\left(Q^z_{kj}-Q^{z-1}_{kj}\right)^2+ \quad (1)$$

$$+\sum_{z=1}^{Z}\sum_{k=0}^{K}\sum_{j=1}^{J_k}\sum_{i=1}^{I_{kji}}C^z_{kji}(q^z_{kji}-q^{Tz}_{kji})^2\to\min.$$

The following restrictions apply:

The relationship between the change in stock of products and the balance of consumption in the sections of lines:

For DL:

$$V^z_{kj}-V^{z-1}_{kj}=\Delta t_z\begin{pmatrix}Q^z_{k,j-1}-\sum_{i=1}^{I_{kj}}q^z_{kji}-Q^z_{kj}-\\-0.5L_{kj}(V^z_{kj}+V^{z-1}_{kj})\end{pmatrix}, \quad (2)$$

$k=\overline{K,1}$, $j=\overline{J_k,1}$, $z=\overline{1,Z}$;

For TL:

$$V^z_{0j}-V^{z-1}_{0j}=\Delta t_z\begin{pmatrix}Q^z_{0,j-1}-\sum_{i=1}^{I_{kj}}q^z_{0ji}-\sum_{k\in K_j}Q^z_{k0}-\\-Q^z_{0j}-0.5L_{0j}(V^z_{0j}+V^{z-1}_{0j})\end{pmatrix}, \quad (3)$$

$j=\overline{J_0,1}$, $z=\overline{1,Z}$;

Here K_j is the set of DL numbers taking products from ML.

Limitations on volumes and expenses of products on the sections of lines:

$$V^{\min}_{kj}\le V^z_{kj}\le V^{\max}_{kj}, k=\overline{0,K}, j=\overline{1,J_k}, z=\overline{1,Z}; \quad (4)$$

$$0\le Q^z_{kj}\le Q^{\max}_{kj}, k=\overline{0,K}, j=\overline{0,J_k}, z=\overline{1,Z}; \quad (5)$$

$$0\le q^z_{kji}\le q^{\max}_{kji}, k=\overline{0,K}, j=\overline{1,J_k}, z=\overline{1,Z}, \quad (6)$$

Problem (1) – (6) is a quadratic programming problem [5]. The constraint (2) and (3) are given in implicit form, and (4), (5) and (6) in explicit form. To determine the possibility of solving this problem, we estimate the number of parameters and limitations. For this purpose, we use the notation:

$$E_1=Z\sum_{k=0}^{K}J_k,$$

$$E_2=E_1+Z\sum_{k=0}^{K}(J_k+1),$$

$$E_3=E=E_2+Z\sum_{k=0}^{K}\sum_{j=1}^{J_k}I_{kj},$$

or $E=Z\sum_{k=0}^{K}J_k+Z\sum_{k=0}^{K}(J_k+1)+Z\sum_{k=0}^{K}\sum_{j=1}^{J_k}I_{kj}$;

$$P_1=Z\sum_{k=1}^{K}J_k, P_2=P_1+ZJ_0,$$

$$P_3=P_2+Z\sum_{k=0}^{K}J_k, P_4=P_3+Z\sum_{k=0}^{K}J_k,$$

$$P_5=P_4+Z\sum_{k=0}^{K}(J_k+1), P_6=P_5+Z\sum_{k=0}^{K}(J_k+1), \quad (7)$$

$$P_7=P_6+Z\sum_{k=0}^{K}\sum_{j=1}^{J_k}I_{kj}, P_8=P=P_7+Z\sum_{k=0}^{K}\sum_{j=1}^{J_k}I_{kj},$$

or $P=5Z\sum_{k=0}^{K}J_k+2Z(K+1)+2Z\sum_{k=0}^{K}\sum_{j=1}^{J_k}I_{kj}$.

In view of (7), we denote

$$x=(x_e^{(1)},x_e^{(2)},x_e^{(3)})=\left\{x_e,\ e=\overline{1,E}\right\}.$$

Here:

$$\begin{aligned} x_e^{(1)} &= V_{kj}^z,\ e=\overline{1,\ E_1}\ ;\\ x_e^{(2)} &= Q_{kj}^z,\ \ e=\overline{E_1+1,\ E_2}\ ;\\ x_e^{(3)} &= q_{kji}^z,\ e=\overline{E_2+1,\ E} \end{aligned} \qquad (8)$$

The restrictions can be written as follows:

$$b=(\,b_p^{(3)},b_p^{(4)},\ b_p^{(5)},\ b_p^{(6)},\ b_p^{(7)},\ b_p^{(8)}).$$

$$\begin{aligned} b_p^{(3)} &= V_{kj}^{\max},\ p=\overline{P_2+1,P_3};\\ b_p^{(4)} &= 0,\ p=\overline{P_3+1,P_4};\\ b_p^{(5)} &= Q_{kj}^{\max},\ p=\overline{P_4+1,P_5};\\ b_p^{(6)} &= 0,\ p=\overline{P_5+1,\ P_6};\\ b_p^{(7)} &= q_{kji}^{\max},\ p=\overline{P_6+1,\ P_7};\\ b_p^{(8)} &= 0,\ p=\overline{P_7+1,\ P}. \end{aligned} \qquad (9)$$

The restrictions can be written as follows:

$$\begin{aligned}
&1)\ \ g_p^{(1)}=V_{kj}^z-V_{kj}^{z-1}-\Delta t_z\left(Q_{k,\,j-1}^z-\sum_{i=1}^{I_{kj}}q_{kji}^z-Q_{kj}^z-0.5L_{kj}(V_{kj}^z+V_{kj}^{z-1})\right)=0,\\
&k=\overline{K,1},\ j=\overline{J_k,1},\ z=\overline{1,Z},\quad p=(z-1)\cdot\sum_{n=K}^{1}J_{n_1}+\sum_{n=K}^{k}J_n+1-j\,,\ p=\overline{1;\ P_1}\ ;\\
&2)\ \ g_p^{(2)}=V_{0j}^z-V_{0j}^{z-1}-\Delta t_z\left(Q_{0,\,j-1}^z-\sum_{i=1}^{I_{0j}}q_{0ji}^z-Q_{0j}^z-\sum_{k\in Kj}Q_{k0}^z-L_{0j}V_{0j}^z\right)=0,\\
&j=\overline{J_0,1},z=\overline{1,Z},\quad p=P_1+(z-1)J_0+J_0+1-j\,,\ \ p=\overline{P_1+1,\ P_2}\,;\\
&3)\ \ g_p^{(3)}=V_{kj}^z\le b_p^{(3)},\quad k=\overline{0,K},\ j=\overline{1,J_k},\ z=\overline{1,\ Z},\\
&p=P_2+(z-1)\sum_{k=0}^{K}J_k+\sum_{n=0}^{k}J_{n-1}+j,\ (J_{-1}=0),\quad p=\overline{P_2+1,\ P_3};\\
&4)\ g_p^{(4)}=-V_{kj}^z\le b_p^{(4)},\quad k=\overline{0,\ K},\ \ j=\overline{1,\ J_k},\ z=\overline{1,\ Z},\\
&p=P_3+(z-1)\sum_{k=0}^{K}J_k+\sum_{n=0}^{k}J_{n-1}+j,\ (J_{-1}=0),\quad p=\overline{P_3+1,\ P_4};\\
&5)\ g_p^{(5)}=Q_{kj}^z\le b_p^{(5)},\quad k=\overline{0,\ K},\ \ j=\overline{0,\ J_k},\ z=\overline{1,\ Z},\\
&p=P_4+(z-1)\sum_{k=0}^{K}(J_k+1)+\sum_{n=0}^{k}(J_{n-1}+1)+1+j,\ (J_{-1}=0),\ \ p=\overline{P_4+1;\ P_5};\\
&6)\ \ g_p^{(6)}=-Q_{kj}^z\le b_p^{(6)},\quad k=\overline{0,\ K},\ \ j=\overline{0,\ J_k},\ z=\overline{1,\ Z},\\
&p=P_5+(z-1)\sum_{k=0}^{K}(J_k+1)+\sum_{n=0}^{k}(J_{n-1}+1)+1+j,\ (J_{-1}=0),\ p=\overline{P_5+1,\ P_6};\\
&7)\ \ \ g_p^{(7)}=q_{kji}^z\le b_p^{(7)},\quad k=\overline{0,\ K},\ \ j=\overline{1,\ J_k},\ \ i=\overline{1,I_{kj}},\ z=\overline{1,\ Z},\\
&p=P_6+(z-1)\sum_{k=0}^{K}\sum_{j=1}^{J_k}I_{kj}+\sum_{n=0}^{k-1}\sum_{m=1}^{J_k}I_{nm}+\sum_{m=1}^{j}I_{k,m-1}+i,\ p=\overline{P_6+1,\ P_7};\\
&8)\ \ g_p^{(8)}=-q_{kji}^z\le b_p^{(8)},\quad k=\overline{0,\ K},\ \ j=\overline{1,\ J_k},\ z=\overline{1,\ Z},\\
&p=P_7+(z-1)\sum_{n=0}^{k}\sum_{j=1}^{J_k}I_{nj}+\sum_{m=1}^{j}I_{k,m-1}+i,\ (I_{k0}=0),\ p=\overline{P_7+1,\ P_8}
\end{aligned} \qquad (10)$$

Given the notation (7), the total number of control parameters and restrictions will be as follows:

$$E=Z\sum_{k=0}^{K}J_k+Z\sum_{k=0}^{K}(J_k+1)+Z\sum_{k=0}^{K}\sum_{j=1}^{J_k}I_{kj}\,,\ \ P=5Z\sum_{k=0}^{K}J_k+2Z(K+1)+Z\sum_{k=0}^{K}\sum_{j=1}^{J_k}I_{kj}\ .$$

Suppose that on each line there are J number of sections and each section has İ number of points of consumption. Then the number of parameters and restrictions will be as follows:

Based on the latest formulas, we obtain the values indicated in table 1.

$$E = Z(K+1)J + Z(K+1)(J+1) + Z(K+1)JI =$$
$$= Z(K+1)(2J+1+JI),$$
$$P = 5Z(K+1)J + 2Z(K+1) + Z(K+1)JI =$$
$$= Z(K+1)(5J+2+JI).$$

From the table 1 it is seen that when using problem (1) – (6) to support systems, the number of control parameters and restrictions is obtained too much. With this in mind, it is necessary to choose or develop as simple and effective a solution to the problem as possible [6].

The solution of the problem

Problem (1) – (6) has been solved for a system consisting of ML and two DL powered by ML. Each line consists of two sections, one consumer in each (Fig. 1).

The control period is divided into three consecutive intervals and the mathematical formulation of the problem is obtained in the following form:

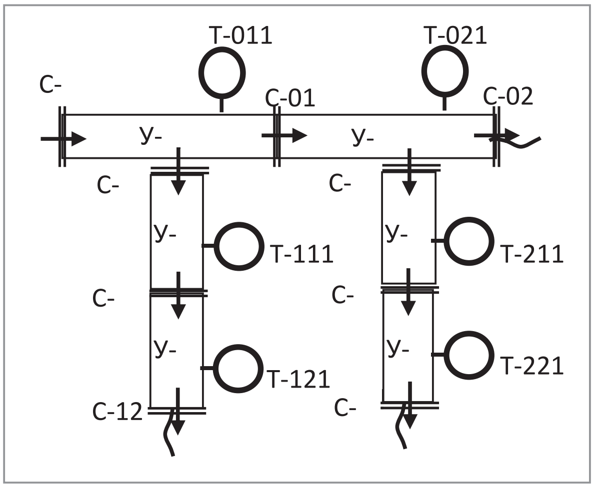

Fig 1.Distributed support system

$$f = \sum_{z=1}^{3}\sum_{k=0}^{2} C_{k2} Q_{k2}^{z} \Delta t_z + \sum_{z=1}^{3}\sum_{k=0}^{2}\sum_{j=1}^{2} C_{kj}^{'z} L_{kj} V_{kj}^{z} + \sum_{z=1}^{3}\sum_{k=0}^{2}\sum_{j=0}^{2} C_{kj}^{z} (Q_{kj}^{z} - Q_{kj}^{z-1})^2 + \sum_{z=1}^{3}\sum_{k=0}^{2}\sum_{j=1}^{2} C_{kji}^{z} (q_{kji}^{z} - q_{kji}^{Tz})^2 \to \min. \quad (11)$$

Table 1. **Number of parameters and restrictions**

Number of time intervals, Z	Number of lines, K	Number of warehouses in one line J	Number of consumers in one line, İ	Number of parameters, E	Number of restrictions P
1	1	1	1	8	16
1	2	2	2	27	48
1	3	3	1	40	80
1	3	3	2	52	92
2	1	1	1	16	32
2	2	2	2	54	96
2	2	3	1	60	120
2	2	3	2	78	138
2	3	3	2	104	184
3	1	1	1	24	48
3	2	2	2	81	144
3	2	3	2	117	207
3	3	3	2	156	276
4	1	1	1	32	64
4	2	2	2	108	192
4	2	3	2	156	276
4	3	3	2	208	368
5	1	1	1	40	80
5	2	2	2	135	240
5	2	3	2	195	345
5	3	3	2	260	460

$$V_{kj}^{z} - V_{kj}^{z-1} = \Delta t_z \begin{pmatrix} Q_{k,j-1}^{z} - q_{kj1}^{z} - Q_{kj}^{z} - \\ -0.5L_{kj}(V_{kj}^{z} + V_{kj}^{z-1}) \end{pmatrix}, \quad (12)$$

$$k = \overline{2,1},\ j = \overline{2,1},\ z = \overline{1.3};$$

$$V_{0j}^{z} - V_{0j}^{z-1} = \Delta t_z \begin{pmatrix} Q_{0,j-1}^{z} - q_{0j1}^{z} - Q_{j0}^{z} - Q_{0j}^{z} - \\ -0.5L_{0j}(V_{0j}^{z} + V_{0j}^{z-1}) \end{pmatrix},$$

$$j = \overline{2,1},\ z = \overline{1.3}; \quad (13)$$

$$V_{kj}^{\min} \le V_{kj}^{z} \le V_{kj}^{\max},$$
$$k = \overline{0,2},\ j = \overline{1,2},\ z = \overline{1,3}; \quad (14)$$

$$0 \le Q_{kj}^{z} \le Q_{kj}^{\max},\ k = \overline{0,2},\ j = \overline{0,2},\ z = \overline{1,3}; \quad (15)$$

$$0 \le q_{kji}^{z} \le q_{kji}^{\max},\ k = \overline{0,2},\ j = \overline{1,2},\ z = \overline{1,3}. \quad (16)$$

From table 1 it can be seen that in problem (11) – (16) the number of variables is 21 (taking into account time intervals the total number of variables is 63), and the number of restrictions is 144. The problem is solved using the direct Hooke-Jeeves method [7], taking into account the restrictions.

Conclusion

From the table 1. shows that when solving the problem at large intervals, the values of some parameters go beyond the boundary values. Therefore, when solving the problem, the intervals of constancy of parameters should be chosen relatively short. But in this case, when solving the problem for all intervals at the same time, the number of parameters and constraints increases significantly. As a result, the solution to the problem is complicated. In order to simplify, the problem can be solved sequentially for each interval and relate the results obtained in real time. In this case, it is necessary to study the deviation of the obtained solutions from their optimal values. Based on the analysis of the results, you can choose the best option for determining the length of time intervals.

REFERENCES

1. **Blackburn, L.** Dynamic optimization of a district energy system with storage using a novel mixed-integer quadratic programming algorithm / L. Blackburn, A. Young, P. Rogers // Optimization And Engineering. – 2019. Vol. 20, № 2. – P. 575–603.
2. **Jabari, F.** Optimal short-term coordination of water-heat-power nexus incorporating plug-in electric vehicles and real-time demand response programs / F. Jabari, B. Mohammadi-ivatloo, J. Ghafouri // Energy. Elsevier, – 2019. – Vol. 174. – P. 708–723.
3. **Гумбатов, И. М.** Разработка автоматизированной системы управления технологическими процессами водозабора и водораспределения (на примере Верхнее-Ханбуланчайского водохозяйственного комплекса) / И. М. Гумбатов // Автореф. Дис. канд. тех. наук. – Баку, – 2006. – 20 с.
4. **Zhuravleva, I. V.** Reconstruction of engineering networks and water supply and wastewater facilities: textbook. manual / I. V. Zhuravleva // Voronezh: Voronezh. state architect. -building. un-t, – 2011. – 146 p.
5. **Искендеров, А. А.** Составление оптимальных графиков распределения воды на разветвленных оросительных каналах / А. А. Искендеров // Научные известия Сумгаитского Государственного Университета, – 2008. – № 2 – С. 85–88.
6. **Melekhov, E. S.** Comprehensive optimization of sources and piping systems for group water supply / E. S. Melekhov // Dis.na sois. uch. Ph.D. on spec. 05.23.04. – Irkutsk, – 2003. – 209 p.
7. **Altinoz, O. T.** Multiobjective Hooke-Jeeves algorithm with a stochastic Newton-Raphson-like step-size method / O. T. Altinoz, A. E. Yilmaz // Expert Systems with Applications. – 2019. – Vol. 117. – P. 166–175. DOI:10.1016/j.eswa.2018.09.033

Поступила *26.06.2020* *После доработки* *08.11.2020* *Принята к печати* *01.12.2020*

ТАГИЕВА А.Д.

РАЗРАБОТКА МОДЕЛИ ОПТИМАЛЬНОГО УПРАВЛЕНИЯ СИСТЕМЫ ВОДООБЕСПЕЧЕНИЯ

Аннотация: В работе рассматривается задача оптимального управления разветвленными системами водообеспечения. Для управления системой разрабатывается задача оптимального распределения продуктов, для решения которой применяется методы решения задач нелинейного программирования. Система состоит из магистральной и распределительных линий. В каждой распределительной линии имеются множество складов. Продукция передается между складами с помощью промежуточных распределительных сооружений. Для устранения недостатков распределения, составлена задача определения необходимых интенсивностей подачи продукции *в сооружениях, позволяющих своевременно обеспечить потребителей необходимым объемом продукции, максимально сократить потери в течение определенного периода.*

Ключевые слова: *Система водообеспечения, водопроводная линия, пункт водопотребления, оптимальное управление, расход воды, критерия оптимизации.*

Тагиева Айгюнь Дамир, Сумгаитский государственный университет, кафедра «Информатики», ст. преподаватель. Азербайджан, г. Баку, пос. Приморск, Tel: +994503950727 irispurpleblack@outlook.com