A. D. TAGHIYEV

DEVELOPMENT OF MODEL OF OPTIMAL CONTROL OF WATER SUPPLY SYSTEM

Abstracts: This paper considers the problem of optimal control of branched water supply systems. To control the system, the problem of optimal distribution of products is developed whereas non-linear programming problems are applied. We consider a system for providing products, consisting of magistral and distribution pipelines, taking products from the magistral pipeline. Each distribution line has many warehouses. Products are taken into the system using the main intake facility and transferred between the warehouses using intermediate distribution facilities. To eliminate the deficiencies in management, tasks are set to determine the necessary intensities of product supply in the facilities, allowing timely provision of consumers with the necessary volume of products, to minimize losses, product discharges of facilities in the system during a certain control period.

Keywords: water supply system, water supply line, water supply point, optimal control, water consumption, optimization criteria.

Introduction

The introduction of modern management systems in the water management provides enterprises with an unprecedented opportunity to control and manage all aspects of water intake, transportation, and distribution from a centralized management system. A new method for optimizing this system in real time is proposed, formulated as an integer quadratic programming problem. The proposed method for solving this problem is very successful in achieving an almost optimal solution. Modern water management enterprises should be a single system, operating with information-computing system [1]. Benefits resulting from these actions can include improving the quality of water supply by reducing water loss, minimizing energy costs, and increasing system performance without compromising operational reliability. The real-time demand management strategy is applied to water supply enterprises to reduce the target cost function as low as possible [2].

Characteristics of the object

In this work, we consider a product supply system, which consists of a magistral line (ML) and K the number of distribution lines (DL) taking products from the ML. In each DL there are \( J_k \) number of warehouses (QW). Products are taken into the system with the help of the main fence (C-00) and transferred between the warehouses with the help of intermediate distribution structures (C-kj, \( k = 0, K \), \( j = 1, J_k \)).

Excessive products can be removed from the system using emergency facilities (C-kJk) at the end of each line. On each j-t section of the q line there are \( l_{kj} \) the number of consumers (T). To control such systems, the necessary intensities of product supply at each structure are calculated, which are supported by an automated control system [3]. But the incorrect calculation of these intensities causes large losses, product discharges, excessive switching of equipment and untimely provision of consumers. As a result of this, the system management efficiency is reduced.

Statement of the problem

To eliminate the above disadvantages, the following statement of the problem of determining the necessary intensities is given:

It is required to find such intensities of supply of products in structures that can provide consumers with the necessary volume of products in a certain manner, minimize losses, product discharges and the number of changes in the operating modes of structures in the system during the control period \( t_0, T \) [4].

We divide the period \( t_0, T \) into Z the number of intervals \( (t_{z-1}, t_z) \), on each of which the intensities are almost constant, \( (t, T) = \bigcup_{z=1}^{Z} (t_{z-1}, t_z) \). We use the following notation: \( y_{kj}^z \), \( k = 0, K \), \( j = 1, J_k \) — reserves of products in warehouses during the interval \( (t_{z-1}, t_z) \); \( Q_{kj}^z \), \( k = 0, K \), \( j = 0, J_k \) — intensities of supply of products in...
line facilities; \( q_{kj}^z, k = 0, K, j = 1, J_k, i = 1, I_{kj} \) — intensities of consumption of products at points of consumption; \( C_{kj}, k = 0, K \) — penalties for dumping a unit volume of production; \( C_{kj}' \) — penalties for losses of a unit volume of production in sections; \( C_{kj}, k = 0, K, j = 0, J_k \), \((kj) \neq (00)\) — penalties for supplying products in intermediate structures with a changed intensity. \( C_{kj}' \) for supplying products with an intensity different from, \( q_{kj}^z \). Product losses in the areas taken proportional to stock products in which \( Q_{kj}^{pot,z} = L_y V_{kj}^z \), \( k = 0, K \), \( j = 0, J_k \), where \( L_{kj} \) — proportionality factor.

As the objective function, the sum of the costs associated with the discharge of products from the automation system \( Q_{kj}^z \), consumption, losses in the areas \( L_y V_{kj}^z \) and, with the squares of the flow rate change in \( C-kj(Q_{kj}^z - Q_{kj}^{-1}) \) and \( T-kj(q_{kj}^z - q_{kj}^{-1}) \):

\[
f = \sum_{z=1}^{Z} \sum_{k=0}^{K} C_{kj} Q_{kj}^z \Delta t_z + \sum_{z=1}^{Z} \sum_{k=0}^{K} \sum_{j=1}^{J_k} C_{kj} L_y V_{kj}^z \Delta t_z + \sum_{z=1}^{Z} \sum_{k=0}^{K} \sum_{j=1}^{J_k} \sum_{i=1}^{I_{kj}} C_{ij} \left[ (Q_{kj}^z - Q_{kj}^{-1}) \Delta t_z \right]^2 + \sum_{z=1}^{Z} \sum_{k=0}^{K} \sum_{j=1}^{J_k} \sum_{i=1}^{I_{kj}} C_{ij} \left[ (q_{kj}^z - q_{kj}^{-1}) \Delta t_z \right]^2 \rightarrow \min.
\]

Denote

\[
C_{00}' = C_{00} = \Delta t_z, C_{kj}' = C_{kj} = \Delta t_z, C_{kj} = C_{kj} = \Delta t_z, C_{kj} = C_{kj} = \Delta t_z.
\]

Then, the objective function is transformed to the form:

\[
f = \sum_{z=1}^{Z} \sum_{k=0}^{K} C_{kj} Q_{kj}^z + \sum_{z=1}^{Z} \sum_{k=0}^{K} \sum_{j=1}^{J_k} C_{kj} Q_{kj}^{pot,z} + \sum_{z=1}^{Z} \sum_{k=0}^{K} \sum_{j=1}^{J_k} \sum_{i=1}^{I_{kj}} C_{ij} \left[ (Q_{kj}^z - Q_{kj}^{-1}) \right]^2 + \sum_{z=1}^{Z} \sum_{k=0}^{K} \sum_{j=1}^{J_k} \sum_{i=1}^{I_{kj}} C_{ij} \left[ (q_{kj}^z - q_{kj}^{-1}) \right]^2 \rightarrow \min.
\]

The following restrictions apply:

The relationship between the change in stock of products and the balance of consumption in the sections of lines:

For DL:

\[
V_{kj}^z - V_{kj}^{-1} = \Delta t_z \left\{ Q_{kj}^z - \sum_{i=1}^{I_{kj}} (q_{kj}^z - q_{kj}^{-1}) \right\} + 0.5 L_y (V_{kj}^z + V_{kj}^{-1}) \tag{2}
\]

\[
k = K_1, j = J_k, z = 1, Z \; ;
\]

For TL:

\[
V_{0j}^z - V_{0j}^{-1} = \Delta t_z \left\{ Q_{0j}^z - \sum_{k=0}^{K} (q_{0j}^z - q_{0j}^{-1}) \right\} - 0.5 L_y (V_{0j}^z + V_{0j}^{-1}) \tag{3}
\]

\[
j = J_0, 1, z = 1, Z \; ;
\]

Here \( K_j \) is the set of DL numbers taking products from ML.

Limitations on volumes and expenses of products on the sections of lines:

\[
V_{kj}^{min} \leq V_{kj}^z \leq V_{kj}^{max}, k = 0, K, j = 1, J_k, z = 1, Z \; ; \tag{4}
\]

\[
0 \leq Q_{kj}^z \leq Q_{kj}^{max}, k = 0, K, j = 0, J_k, z = 1, Z \; ; \tag{5}
\]

\[
0 \leq q_{kj}^z \leq q_{kj}^{max}, k = 0, K, j = 1, J_k, z = 1, Z \; . \tag{6}
\]

Problem (1) – (6) is a quadratic programming problem [5]. The constraint (2) and (3) are given in implicit form, and (4), (5) and (6) in explicit form. To determine the possibility of solving this problem, we estimate the number of parameters and limitations. For this purpose, we use the notation:

\[
E_1 = Z \sum_{k=0}^{K} J_k, \quad E_2 = E_1 + Z \sum_{k=0}^{K} (J_k + 1), \quad E_3 = E_2 + Z \sum_{k=0}^{K} \sum_{j=1}^{J_k} I_{kj}, \quad \text{or} \quad E = Z \sum_{k=0}^{K} J_k + Z \sum_{k=0}^{K} (J_k + 1) + Z \sum_{k=0}^{K} \sum_{j=1}^{J_k} I_{kj} ;
\]

\[
P_1 = Z \sum_{k=0}^{K} J_k, P_2 = P_1 + Z J_0, \quad P_3 = P_2 + Z \sum_{k=0}^{K} J_k, P_4 = P_3 + Z \sum_{k=0}^{K} J_k, \quad P_5 = P_4 + Z \sum_{k=0}^{K} (J_k + 1), P_6 = P_5 + Z \sum_{k=0}^{K} (J_k + 1), \quad (7)
\]

\[
P_7 = P_6 + Z \sum_{k=0}^{K} \sum_{j=1}^{J_k} I_{kj}, P_8 = P_7 + Z \sum_{k=0}^{K} \sum_{j=1}^{J_k} I_{kj}, \quad \text{or} \quad P = 5Z \sum_{k=0}^{K} J_k + 2Z (K + 1) + 2Z \sum_{k=0}^{K} \sum_{j=1}^{J_k} I_{kj} .
\]
In view of (7), we denote
\[ x = (x_e^{(1)}, x_e^{(2)}, x_e^{(3)}) = \left\{ x_e, \ e = 1, E \right\}. \]

Here:
\[ x_e^{(1)} = V_{b,1}^z, \ e = 1, E; \]
\[ x_e^{(2)} = Q_{b,1}^z, \ e = E_1 + 1, E_2; \]
\[ x_e^{(3)} = q_{b,1}^z, \ e = E_2 + 1, E. \]

The restrictions can be written as follows:
\[ b = (b_p^{(3)}, b_p^{(4)}, b_p^{(5)}, b_p^{(6)}, b_p^{(7)}, b_p^{(8)}). \]

The restrictions can be written as follows:
\begin{align*}
1) \quad & g_p^{(1)} = V_{kj}^z - V_{kj}^{z-1} - \Delta t_z \left( Q_{b,1}^z - \sum_{i=1}^{k} q_{b,1}^z - Q_{b,1}^z - 0.5 L_{b,1}^z (V_{kj}^z + V_{kj}^{z-1}) \right) = 0, \\
& k = K, j = J, z = Z, \quad p = (z-1) \cdot \sum_{n=k}^{1} J_n + \sum_{n=k}^{1} J_n - 1 + j, \quad p = 1, P_1; \\
2) \quad & g_p^{(2)} = V_{kj}^z - V_{kj}^{z-1} - \Delta t_z \left( Q_{b,1}^z - \sum_{i=1}^{k} q_{b,1}^z - Q_{b,1}^z - \sum_{k=1}^{k} Q_{b,1}^z - L_{b,1}^z V_{kj}^z \right) = 0, \\
& j = J, z = Z, \quad p = P_1 + (z-1) J_0 + J_0 + 1 + j, \quad p = P_1 + P_2; \\
3) \quad & g_p^{(3)} = V_{kj}^z \leq b_p^{(3)}, \quad k = 0, K, \quad j = 1, J, \quad z = 1, Z, \\
& p = P_2 + (z-1) \sum_{k=0}^{K} J_k + \sum_{n=0}^{k} J_{n-1} + j, \quad (J_1 = 0), \quad p = P_2 + 1, P_3; \\
4) \quad & g_p^{(4)} = -V_{kj}^z \leq b_p^{(4)}, \quad k = 0, K, \quad j = 1, J, \quad z = 1, Z, \\
& p = P_3 + (z-1) \sum_{k=0}^{K} J_k + \sum_{n=0}^{k} J_{n-1} + j, \quad (J_1 = 0), \quad p = P_3 + 1, P_4; \\
5) \quad & g_p^{(5)} = Q_{b,1}^z \leq b_p^{(5)}, \quad k = 0, K, \quad j = 0, J, \quad z = 1, Z, \\
& p = P_4 + (z-1) \sum_{k=0}^{K} (J_k + 1) + \sum_{n=0}^{k} (J_{n-1} + 1) + j, \quad (J_1 = 0), \quad p = P_4 + 1, P_5; \\
6) \quad & g_p^{(6)} = -Q_{b,1}^z \leq b_p^{(6)}, \quad k = 0, K, \quad j = 0, J, \quad z = 1, Z, \\
& p = P_5 + (z-1) \sum_{k=0}^{K} (J_k + 1) + \sum_{n=0}^{k} (J_{n-1} + 1) + j, \quad (J_1 = 0), \quad p = P_5 + 1, P_6; \\
7) \quad & g_p^{(7)} = q_{b,1}^z \leq b_p^{(7)}, \quad k = 0, K, \quad j = 1, J, \quad i = 1, I, \quad z = 1, Z, \\
& p = P_6 + (z-1) \sum_{k=0}^{K} \sum_{j=1}^{J_k} I_{kj} + \sum_{n=0}^{k} \sum_{j=1}^{J_n} I_{nj} + \sum_{m=1}^{J_{m-1}} I_{km} + i, \quad p = P_6 + 1, P_7; \\
8) \quad & g_p^{(8)} = -q_{b,1}^z \leq b_p^{(8)}, \quad k = 0, K, \quad j = 1, J, \quad z = 1, Z, \\
& p = P_7 + (z-1) \sum_{k=0}^{K} \sum_{j=1}^{J_k} I_{kj} + \sum_{m=1}^{J_{m-1}} I_{km} + i, \quad (I_{k,0} = 0), \quad p = P_7 + 1, P_8.
\end{align*}

Given the notation (7), the total number of control parameters and restrictions will be as follows:
\[ E = Z \sum_{k=0}^{K} J_k + Z \sum_{k=0}^{K} (J_k + 1) + Z \sum_{k=0}^{K} \sum_{j=1}^{J_k} I_{kj}, \quad P = 5Z \sum_{k=0}^{K} J_k + 2Z (K + 1) + Z \sum_{k=0}^{K} \sum_{j=1}^{J_k} I_{kj}. \]
Suppose that on each line there are $J$ number of sections and each section has $I$ number of points of consumption. Then the number of parameters and restrictions will be as follows:

Based on the latest formulas, we obtain the values indicated in table 1.

$$E = Z(K + 1)J + Z(K + 1)(J + 1) + Z(K + 1)JI = Z(K + 1)(2J + 1 + JI),$$

$$P = 5Z(K + 1)J + 2Z(K + 1) + Z(K + 1)JI = Z(K + 1)(5J + 2 + JI).$$

From the table 1 it is seen that when using problem (1) – (6) to support systems, the number of control parameters and restrictions is obtained too much. With this in mind, it is necessary to choose or develop as simple and effective a solution to the problem as possible [6].

**The solution of the problem**

Problem (1) – (6) has been solved for a system consisting of ML and two DL powered by ML. Each line consists of two sections, one consumer in each (Fig. 1).

The control period is divided into three consecutive intervals and the mathematical formulation of the problem is obtained in the following form:

$$f = \sum_{z=1}^{3} \sum_{k=0}^{2} C_{k}^{z} Q_{k}^{z} \Delta t_{z} + \sum_{z=1}^{3} \sum_{k=0}^{2} \sum_{j=1}^{2} C_{kj}^{z} L_{kj} V_{kj}^{z} + \sum_{z=1}^{3} \sum_{k=0}^{2} \sum_{j=1}^{2} C_{j}^{z} (Q_{kj}^{z} - Q_{kj}^{z-1})^{2} + \sum_{z=1}^{3} \sum_{k=0}^{2} \sum_{j=1}^{2} C_{k}^{z} (q_{kj}^{z} - q_{kj}^{z-1})^{2} \rightarrow \min.$$  

(11)

**Table 1. Number of parameters and restrictions**

<table>
<thead>
<tr>
<th>Number of time intervals, $Z$</th>
<th>Number of lines, $K$</th>
<th>Number of warehouses in one line $J$</th>
<th>Number of consumers in one line, $I$</th>
<th>Number of parameters, $E$</th>
<th>Number of restrictions $P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>27</td>
<td>48</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>40</td>
<td>80</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>16</td>
<td>32</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>54</td>
<td>96</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>60</td>
<td>120</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>78</td>
<td>138</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>104</td>
<td>184</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>24</td>
<td>48</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>81</td>
<td>144</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>117</td>
<td>207</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>156</td>
<td>276</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>32</td>
<td>64</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>108</td>
<td>192</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>156</td>
<td>276</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>208</td>
<td>368</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>40</td>
<td>80</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>135</td>
<td>240</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>195</td>
<td>345</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>260</td>
<td>460</td>
</tr>
</tbody>
</table>
В работе рассматривается задача оптимального управления разветвленными системами водообеспечения. Для управления системой разрабатывается задача оптимального распределения продуктов, для решения которой применяется метод решения задач неллинейного программирования. Система состоит из магистральных и распределительных линий. В каждой распределительной линии имеется множество складов. Продукция передается между складами с помощью промежуточных распределительных сооружений. Для устранения недостатков распределения, составлена задача определения необходимых интенсивностей подачи продукции,

$$V_{kj} - V_{kj}^{-1} = \Delta z \left( Q_{kj}^- - q_{kj}^- - Q_{kj}^+ - 0.5L_{kj}(V_{kj}^+ + V_{kj}^-) \right),$$

$$V_{0j} - V_{0j}^{-1} = \Delta z \left( Q_{0j}^- - q_{0j}^- - Q_{0j}^+ - 0.5L_{0j}(V_{0j}^+ + V_{0j}^-) \right),$$

$$0 \leq Q_{kj}^j \leq Q_{kj}^\max, \quad k = 0, 2, \quad j = 1, 2, \quad z = 1, 3;$$

$$0 \leq q_{kj}^j \leq q_{kj}^\max, \quad k = 0, 2, \quad j = 1, 2, \quad z = 1, 3.$$
в сооружениях, позволяющих своевременно обеспечить потребителей необходимым объемом продукции, максимально сократить потери в течение определенного периода.

**Ключевые слова:** Система водообеспечения, водопроводная линия, пункт водопотребления, оптимальное управление, расход воды, критерия оптимизации.

Тагиева Айгюнь Дамир, Сумгaitский государственный университет, кафедра «Информатики», ст. преподаватель. Азербайджан, г. Баку, пос. Приморск, Tel: +994503950727 irispurpleblack@outlook.com